DIVINE LOGIC

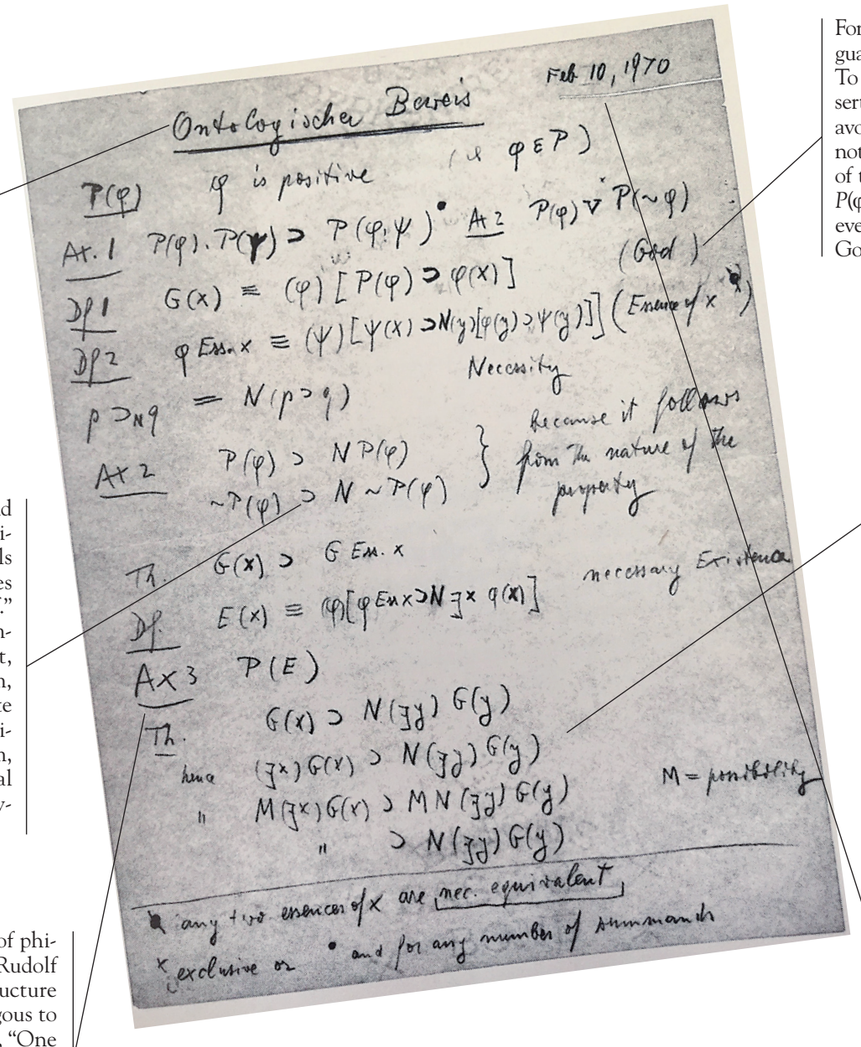
Kurt Gödel's last proof

By Stephen Budiansky

At the age of twenty-four, the Austrian-born logician Kurt Gödel stunned the world of mathematics with his incompleteness theorem, which demonstrated that the field must allow for statements that are true but unprovable. In the decades following that precocious feat, he worked on more obscure philosophical explorations that were to remain unpublished in his lifetime. "Philosophy," he once complained to his friend the economist Oskar Morgenstern, "is today—at best!—where the Babylonians were with mathematics." In 1970, he revealed one of these exercises to a few colleagues: a proof for the existence of God. Like his philosophical hero, Gottfried Leibniz, Gödel called it *Ontologischer Beweis*—an ontological proof, one that derives God from abstract reason rather than empirical evidence. He told Morgenstern that he hesitated to publish his result lest people think that he "really does believe in God," insisting he was merely carrying out a "logical investigation." But in letters to his mother and in conversations with friends, Gödel expressed his conviction that a scientific proof of "the theological worldview" would imply not only the existence of God but also of an afterlife.

Gödel's investigations built on the pioneering work of Gottlob Frege and Bertrand Russell, who, at the turn of the twentieth century, developed systems for symbolically expressing and deriving logical propositions. Much as algebra uses symbols like *x* and *y* to represent variables in arithmetical operations, symbolic logic uses symbols such as ⊃ to represent "if–then," ~ for "not," and ≡ for "if and only if." Russell hoped to show that all of mathematics could be derived from a small number of purely logical propositions so self-evidently true as to be beyond doubt, thereby securing the foundations of the discipline. In his incompleteness theorem, Gödel turned that expectation on its head, using logic and arithmetic to formulate a statement which asserts that it *cannot* be derived from any of the basic propositions of this system. To him, this was strong confirmation of mathematical realism, the view that mathematical entities such as numbers exist in some ways external to the human mind—that humans did not create mathematics, but only discovered preexisting mathematical truths.

Gödel had raised the idea of applying pure logic to the most profound questions of philosophy and theology as early as 1940, in a conversation with the philosopher Rudolf Carnap. His fundamental notion was to define certain axioms about the logical structure of the world—these are the statements labeled "Ax" in the proof—that are analogous to self-evident truths in mathematics, such as that zero equals zero. As he told Carnap, "One could establish an exact system of postulates employing concepts that are usually considered metaphysical: 'God,' 'soul,' 'idea.' If this is done accurately, there would be no objection." Gödel argued that just as Newton laid the foundations of modern physics by defining a few "primitive" concepts such as mass and force, it ought to be similarly possible to establish a philosophical truth by identifying a few primitive entities that cause the existence of everything else.



For his ontological proof, Gödel introduced a property he called "God-likeness." In the language of symbolic logic, φ(*x*) stands for the statement, "The object *x* possesses the property φ." To formulate the statement "*x* is God-like," which he abbreviates here as G(*x*), Gödel first asserted his axiom that any property an object can possess is either "positive" or "negative." He avoided defining these terms precisely, but explained that by "positive" he meant something not simply good, but "purely good," embodying a sense of inherent perfection "independently of the accidental structure of the world." Then, using (φ) to mean "for every property φ" and *P*(φ) to mean "the property φ is a positive property," Gödel wrote a formula that asserts: For every property φ, if φ is a positive property, then *x* possesses that property. In other words, a God-like object has every possible positive property and zero negative properties.

The crux of Gödel's proof was demonstrating that by combining that definition of God-likeness with the rules of logical inference, plus a few other basic assumptions—including the proposition that existence itself is a positive property—one could show that "the system of all positive properties" does not lead to any contradictions: in other words, that a God-like object can exist. The final step was to prove that if such an object *can* exist, it *must* exist. Gödel acknowledged that these ideas—even his interest in these questions—placed him squarely outside the mainstream of contemporary philosophical and scientific thinking. ("I *don't* consider my work a 'facet of the intellectual atmosphere of the early 20th century,' but rather the opposite," he once wrote.) A fellow logician, Gerald Sacks, recalled Gödel's almost childlike enthusiasm in extracting profound truth from simple logic: he once described to Sacks an elementary proof showing that it was impossible to define in language the philosophical notion of the Absolute, since to do so would defy the rules of set theory. "Then his eyes lit up," Sacks recalled, "and he said, 'Isn't that wonderful, we know something important about the Absolute, just by logic.'"

The date of this proof—February 10, 1970—carries an ominous significance. That same day, irrationally convinced that he was either going to die or be declared mentally incompetent, he told Morgenstern the location of his unpublished works so that his friend could arrange for their posthumous publication. Gödel had exhibited previous episodes of severe paranoia, interspersed with longer stretches of lucidity and cheerful productivity. But as revealed in his diaries, he was never free from deep anxieties about life and the opinions of others, and carried with him the sense of not having lived up to his early promise. "Observation by others is embarrassing to you," he wrote, "*because you believe they expect something grand.*" Obsessed with his digestion and beset by hypochondria and fears of being poisoned, he hardly ate. By the spring of 1970, he began hallucinating. He told Morgenstern and his psychiatrist that the Nazis were after him, that he was about to be fired by the Institute for Advanced Study, that people hated him, and that doctors were lying to him about his condition. He temporarily recovered, only to plunge deeper into unreality in the summer of 1977. "A tragedy and how in the world can I help him," Morgenstern wrote two weeks before his own death from cancer that July. Six months later, Gödel was dead, having succumbed at last to his years of slow starvation. ■

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Kurt Gödel Papers, Shelby White and Leon Levy Archives Center, Institute for Advanced Study, Princeton University Library